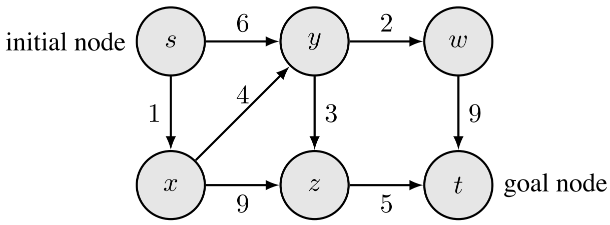
3010 Artificial Intelligence — Assignment 1
Due: Tuesday, May 21, 2019

Write a report answering Questions 1–4 (Note: questions continue to the back of the page). Submit the report in the drop-in box in front of Information Science Administration Office, by no later than May 21.

Question 1

Consider the state space graph shown below. This graph has six nodes $\{s, t, w, x, y, z\}$, among which s is the initial node and t is the goal node. The digit next to each arc represents the cost of the arc. For instance, the cost of arc (x, z) is $c(x, z) = 9$. Answer the following questions.



1. Find the cost of the cheapest path from ***each*** of the six nodes to the goal node t.
2. Suppose we run on this graph Dijkstra's shortest-path algorithm shown in Figure 1. In each iteration of lines 6–11 of function Dijkstra (in Figure 1),
   - show which nodes are in OPEN and what are their g-values when line 7 is executed, and
   - show which node is chosen as v on line 8.

```
1  function Dijkstra(s)
2      OPEN ← new PriorityQueue_g
3      g[s] ← 0
4      Insert_g(OPEN, s)
5      CLOSED ← ∅
6      loop do
7          if IsEmpty(OPEN) then return "failure"
8          v ← DeleteMin_g(OPEN)
9          CLOSED ← CLOSED ∪ {v}
10         if IsGoal(v) then return Solution(v, s)
11         Expand(v)
```

```
1  procedure Expand(v)
2      foreach u ∈ Succ(v) do
3          if u ∉ OPEN ∪ CLOSED then
4              Parent[u] ← v
5              g[u] ← g[v] + c(v, u)
6              Insert_g(OPEN, u)
7          else if u ∈ OPEN then
8              if g[v] + c(v, u) < g[u] then
9                  Parent[u] ← v
10                 g[u] ← g[v] + c(v, u)
```

Figure 1: Dijkstra's shortest path algorithm. OPEN, CLOSED, Parent, and g are global variables. See the lecture slides for other details.

Question 2

Draw a state space graph G that satisfies all of the following conditions:

1. G has four nodes $\{s, t, x, y\}$, where s is the initial node and t is the (only) goal node;
2. G has six edges;
3. all edge costs are positive integers less than or equal to 6; and
4. when Dijkstra's shortest-path algorithm (of Figure 1) is run on G, the g value for one node is changed twice (i.e., line 10 of procedure Expand is executed twice).

Question 3

Let v_j be the jth node closed (i.e., placed in the CLOSED set) during a run of Dijkstra's algorithm, $j = 1, 2, \ldots$. (Because the first node closed by the algorithm is the initial node s, $v_1 = s$). For any node v, let $g^*(v)$ denote the cost of the chepeast path from the initial node s to v. Prove that $g^*(v_j)$ is nondecreasing over j; that is, $g^*(v_1) \leq g^*(v_2) \leq g^*(v_3) \leq \cdots$.

Question 4

Consider a finite state space graph with (only) one goal node. Modify Dijkstra's algorithm so that when run on such a graph, it returns the ***number*** of cheapest paths from the initial node to the goal node, instead of a cheapest path to the node.

For example, in the following state space graph with the initial node s and the goal node t, the modified algorithm must return 4, because there are four cheapest paths (all with cost 5), namely, (1) $s \to a \to c \to d \to t$, (2) $s \to a \to c \to e \to t$, (3) $s \to b \to c \to d \to t$, and (4) $s \to b \to c \to e \to t$. Note: cheapest paths need not be enumerated; only their number (i.e., "4" in this example) must be output.